Trading Option Portfolios using Combinatorial Exchange

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Abstract. In finance, one of the cornerstones of pricing any derivative contract is the no-arbitrage theory where derivatives such as options are priced according to the risk-neutral measure using the seminal Black-Scholes formula. However this model disregards the market sentiment expressed through investor's demand or supply on such derivatives. Option portfolios such as bullish, bearish or butterfly spreads are commonly traded bundles of options which signal the trader's prediction on the future price of an underlying asset. In this paper, we depart from the traditional no-arbitrage principles of option pricing and apply Parkes et al. Iterative Combinatorial Exchange (ICE) for trading option portfolios where traders can express their market sentiment via their combinatorial preferences for different options. We express different option portfolios using Tree-based Bidding Language (TBBL) and use corresponding Winner Determination Problem (WDP) along with the Threshold payment rule to compute the clearing prices for any given option portfolio. Moreover, we extend TBBL to incorporate spreads of all types, so the responsibility of compiling an option portfolio shifts to a mechanism rather than to a trader.

Keywords: Application of Combinatorial Exchanges, Bidding Languages, Financial Markets and Derivatives

1 Introduction

In finance, option is a contract that gives the right of buying (selling) certain asset at an agreed future price (i.e. *strike price*) to its holder. The issuer of such contract charges a premium from the holder to compensate the future liability that he may incur. Holder of the option can exercise his right on a agreed period of time. For example, European options can be exercised only on their maturity date, while American options on any date until the expiration of the contract. We will use only European options in the scope of this paper.

Options constitute significant part of modern financial trades, and are priced according to the risk associated with buying or selling an underlying asset in future. In finance literature such risk is evaluated from an arbitrage-free perspective where no financial contract can yield riskless profit to its holder. The seminal works by Black and Scholes [2], and Merton[9] established a solid framework for pricing derivative contracts such as options and derived a closed-form solution for computing the risk-neutral value of any European option. However, according to the survey of empirical data and the contemporary methods of option pricing accomplished by Bates[1], the parametric models such as Black-Scholes formula cannot fully capture the "financial inter-mediation of the underlying risks by option market-makers". In Black-Scholes, options are priced with a constant volatility (variance) of an underlying asset in mind. However, empirically speaking, the emergence of a volatility curve for options suggests the opposite where implied volatility is changing and options are valued differently depending on their moneyness (Out-of-The-Money (OTM), In-The-Money (ITM) and At-The-Money (ATM)) options. Most of finance literature models this phenomenon via stochastic variance [7] or via other frictions and jumps in the market. This would lead us to an idea of pricing options in such way which would cover the whole spectrum of option moneyness in single process. This becomes possible when traders can express their combinatorial preferences for option portfolios spanning options from different moneyness range and obtain competitive equilibrium prices for them through a combinatorial exchange.

The ICE presented by Parkes et al. is an expressive and a complete mechanism stemming from earlier works accomplished in combinatorial auctions [5] involving such concepts as price discovery and activity rules. Its bidding language TBBL generalises the expressiveness and completeness of multiple bidding languages such as `OR`, `XOR`, etc proposed earlier for combinatorial auctions [10]. Its efficiency can be asymptotically approximated with the number of goods or traders participating in the mechanism. Its payment rule is budget-balanced, and most importantly Dominant Strategy Incentive Compatible (DSIC) and individual rational. However similar to combinatorial auctions, its WDP is NP-hard as it can be directly transformed into set packaging problem.

The paper shows the novel application of combinatorial exchanges in the market of financial derivatives, namely, options. We use Parkes et al. [11] ICE as a reference model for the design of a combinatorial exchange and use its TBBL [3] to enable the structured bidding of option portfolios. We define cases where options could be seen as complements and substitutes, and on the basis of this definition use combinatorial exchange to evaluate their prices. We also provide examples on different ways of expressing commonly known option portfolios using TBBL. Moreover, we suggested using TBBL for expressing the generic structure of different types of option portfolios such as spreads, ladders, straddles, etc. By expressing the option preferences in a generic way, traders can shift the burden of compiling the most efficient option portfolios matching with their choice of option portfolio type to the mechanism itself thanks to direct revelation principle. On the other hand, the mechanism will satisfy more combinatorial orders because of its freedom to match options with different strike but with the same moneyness.

2 Preliminaries

Option's value depends on several parameters of the underlying market and the conditions written in the contract such as strike price K and asset's spot price S_t. Option belongs to different *moneyness* range depending on if its strike is greater or less than the current asset price. Put option (i.e. an option that gives right to sell at an agreed price in future) is said to be if its strike price is below the market's price, if it is above the market price and if it is equal to the market price. Call option (i.e. an option that gives right to buy at an agreed price in future) is said to be Out-of-The-Money (OTM) if its strike price is above the market's price, In-The-Money (ITM) if it is below the market price and At-The-Money (ATM) if it is equal to the market price. Table 1 summarises the options by moneyness.

Table 1: Options by Moneyness

	OTM	ATM	ITM
CALL	$K > S_t$	$K = S_t$	$K < S_t$
PUT	$K < S_t$	$K = S_t$	$K > S_t$

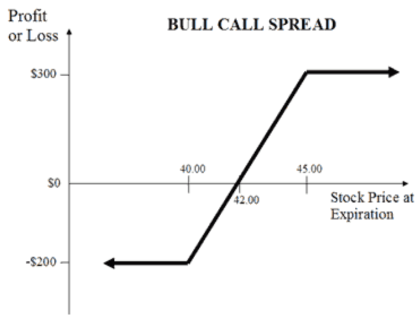

Fig. 1: Bull Spread with Call Options

Most of the option portfolios, except calendar option portfolios, are made using options with different moneyness to cut the risk of infinite loss, and take advantage of agent's future forecast. For example, if bullish trader hopes that the asset price will increase. Therefore he buys one ITM call option at c_1 with strike price K_1 and sells one OTM call option c_2 with higher strike price K_2. Both options have same expiration date. Because K_2 is higher than K_1, c_2 should be lower than c_1. If the asset price becomes less than K_1, trader will loose $c_2 - c_1$. If the asset price falls between K_1 and K_2, then trader's payoff is the difference between current price S_T and K_1 minus the difference in option prices. Bullish trader wants the asset price to go up, so he can make fixed profit of $K_2 - K_1$ minus the difference in option prices. The same bull spread can be made with put options as well, so the trader buys put with low strike price and sells another put with high strike price. Payoff for bull spread with call options is written below (1):

$$P = \begin{cases} c_1 - c_2 & \text{if } S_T \leq K_1 \\ S_T - K_1 - (c_1 - c_2) & \text{if } K_1 < S_T < K_2 \\ K_2 - K_1 - (c_1 - c_2) & \text{if } S_T \geq K_2 \end{cases} \tag{1}$$

where $c_1 > c_2$ and $K_1 < K_2$. Figure 1 illustrates bull spread with call options.

3 Substitutability and Complementarity of Options

The substitutability and complementarity of options are not defined in financial literature. In a risk-neutral and frictionless world, option is a contract which mimics the payoff function of a delta-hedged portfolio. Therefore its value is nothing but the value of that replicating portfolio which consists of cash and assets [8]. As long as the cash and assets are identical and anonymously priced, the options on top of them can be priced according to the same parameters which are invariant. This would mean that every option has uniquely defined price which is risk-neutral compared to the conditions of the underlying market. Once the supply and the demand are not involved in pricing the options, the economic concepts such as cross-demand (cross-supply) elasticity XDE (XSE) which define substitutability (i.e XDE is positive) and complementarity (i.e. XDE is negative) of goods are also disregarded in option market. However as we introduced the market component into the option pricing methodology, and determined the option prices using the supply and the demand for options, then the question of XDE and XSE may naturally arise in studying the substitutability and complementarity relationships between different options.

For example, let us consider a case where options can be viewed as substitutes. There is a bearish sentiment (i.e. the asset price is expected to fall) in underlying market and majority of option traders wish to take bearish spread which consists of buying one OTM call and selling one ITM call. However traders have multiple choices for OTM calls with different strikes. If over-demand is assumed to be the main reason for the rise of OTM call prices, then for some OTM if the price remains the same, the traders would be willing to buy it at cheaper price compared to other OTMs. This, as a result, would increase the demand for this option. Hence OTM options are substitutes in bearish spread.

Another aspect of goods as substitutes is the unwillingness of the traders buying two substitute items simultaneously. Using our previous example, trader willing to take bearish spread does not want to buy two OTM options having sold only one ITM. This would distort his option portfolio and change its payoff function. In given example, trader can have a valuation as $v(\{OTM1, OTM2\}) = v(\{OTM1\}) \oplus v(\{OTM2\})$, because he wants to buy only one OTM, so he pays only for one option allocated to him and takes the second one for free. Let us assume that out of multiple OTMs allocated to the trader, it is for the cheapest OTM he pays. So his valuation function is, indeed, $v(\{OTM1, OTM2\}) = \min[v(\{OTM1\}), v(\{OTM2\})]$. This is definitely smaller than their combined valuation, so according to the definition of substitutability given below (2), these two options OTM1 and OTM2 are substitutable for the trader who wants to take a bear spread.

$$v(\{OTM1, OTM2\}) \leq v(\{OTM1\}) + v(\{OTM2\}) \tag{2}$$

In order to understand the complementarity of options, let us consider the trader who wants to take bullish spread. In this case, the trader needs to buy one ITM call, and sell one OTM call. We know that the ITM costs more than OTM, so the trader's linear price of this contract is $v(\{ITM, -OTM\}) = \hat{v}(\{ITM\}) - \hat{v}(\{OTM\}) \geq 0$[1]. However the trader is determined to take bullish spread, and considers no other choice. So this would mean that if the trader gets allocated with either one of the options, but he is refused for the other one, his bid for ITM is zero, and ask for OTM is infinity. In other words, both $v(\{ITM\}) = 0$ and $v(\{-OTM\}) = \infty$ is true. Hence we know that $v(\{ITM\}) - v(\{-OTM\}) < 0$, we can derive an inequality given below which defines the strong complementarity of goods.

$$v(\{ITM, -OTM\}) > v(\{ITM\}) - v(\{-OTM\}) \tag{3}$$

[1] Minus sign means short position. $\hat{v}(\cdot)$ is the intrinsic value of the option for the trader, not the reported one.

To sum up, the traders engaged in taking option portfolios may exhibit a valuation behaviour which makes OTM options (and ITM options by symmetry) with different strikes mutually substitutable. Also we have seen that ITM and OTM options could be complements to each other when they are wanted as an integral part of an option portfolio. Once we can establish such relationships between options, there is an immediate necessity for a combinatorial exchange which can accept such preferences from traders and efficiently allocate them to winning traders. We propose a design of such combinatorial exchange that can deal with the substitutability and complementarity of options inside option portfolios.

4 Design of Combinatorial Exchange

There are two ways of looking at option market for traders using option portfolios. First way is looking at the market as a multi-unit multi-item double auction where traders simply submit their orders for each option separately and then once their orders satisfied, they hold a certain option portfolio. We can also consider this case as simultaneous multi-unit double auctions run in parallel for different types of options. Cramton [5] describes an issue for simultaneously ascending auction where traders had an incentive to snipe in an auction which bid the least price. In order to prevent bidders engaging in sniping, the auction introduced set of activity rules into its protocol. One of them was not to allow bidders increase their volumes as the price goes up, as it contradicts to the law of demand. Not imposing such activity rule would allow bidders to put a bid with an insignificant volume to stay active in multiple simultaneous auctions until the last moment when the trader put all his required volume into the cheapest auction and wins the lot. Once the activity rules are applied, the mechanism can produce surplus maximised outcome and hence be efficient.

However, the second way of dealing with option portfolios is matching them in a combinatorial exchange. In this way, traders may not reveal their individual valuations of the options to the mechanism. Also traders are not required to coordinate their bids and asks in multiple auctions to make sure that their option portfolio is compiled. In more general perspective, combinatorial exchange allows the traders to express much more information other than simple quotes on options or option portfolios they want. In fact, in combinatorial exchange traders can reveal their whole strategy, or bearish or bullish beliefs, to allow the mechanism to decide which option portfolio one needs in order to maximise his utility. In combinatorial exchanges the problem of finding best allocation and computing DSIC payment rule can be translated into set packaging problem which is NP-hard. This problem can be easily tackled with simultaneous multi-unit double auctions (SMUDA) if goods-are-substitutes because the surplus maximised allocation of goods in SMUDA sets the upper-bound for a combinatorial exchange [12]. However the combinatorial exchange allocation becomes NP-hard when the goods-are-complements, and SMUDA cannot produce better surplus maximisation. With this outcome, the implementation of a combinatorial exchange for option portfolios can be justified.

We propose the design of a combinatorial exchange for trading option portfolios using Parkes et al. [11] ICE as a reference model. We use TBBL to specify the preferences of option traders. Then we formulate the WDP and payment rules. Let us specify the notation we used for denoting traders, options, allocations etc:

$N = \{1, \ldots, n\}$ is the set of traders
$G = \{1, \ldots, m\}$ is the option types listed in the option chain
$x^0 = (x_1^0, \ldots, x_n^0)$ is the the limits for traders in selling options. $x_i^0 = (x_{i1}^0, \ldots, x_{im}^0)$ where $x_{ij}^0 \in \mathbb{Z}_+$ is the limit for each trader to sell particular type of option. This limit can be imposed by the margin account of the trader, or by the company's policy, or as we did in previous chapters, by the mechanism itself. Hence these limits are the quantity cap for the maximum number of options a trader can sell.
$\lambda = (\lambda_1, \ldots, \lambda_n)$ denotes the change in the allocation of options (i.e. trade), while each $\lambda_i = (\lambda_{i1}, \ldots, \lambda_{im})$ and $\lambda_{ij} \in \mathbb{Z}$. So λ_{ij} is the change in agent i's jth option account, negative number meaning the sales, and positive the acquisition.
$M = \sum_{i \in N} \sum_{j \in G} x_{ij}^0$ is the maximum number of options that could be traded in the combinatorial exchange.

4.1 Efficient Trades

For each possible trade for the agent i, we can define valuation function $v_i(\lambda_i) \in \mathbb{R}$ which denote how much the trader is willing to pay or receive for given set of trades in λ_i. Using our previous example, if the trader wants bearish spread only, then his valuation for OTM options should be $v_i(\lambda_{OTM}) = \inf_{\lambda_{ij}>0, j \in \lambda_{OTM}}(v_i(\lambda_{ij}))$, as he considers the OTM options as substitutes. Let us denote the final position for the trader i as $x_i^0 + \lambda_i \geq 0$ which would mean that the agent's capacity decreases as he sells more options, and it should not exceed the given cap x_i^0.

We use the *free disposal* assumption so $v_i(\lambda_i') \geq v_i(\lambda_i) \rightarrow \lambda_i' \geq \lambda_i, \forall j \quad \lambda_{ij}' \geq \lambda_{ij}$ is true. Also let us denote the overall surplus from the trade as $v(\lambda) = \sum_i v_i(\lambda_i)$. Traders use quasi-linear utility to evaluate each trader: $u_i(\lambda_i, p) = v_i(\lambda_i) - p$ where p is the price paid for the trade λ_i. The quasi-linearity of utilities guarantee that any Pareto improvement to the allocation maximises the social surplus, so for the given profile of valuations and caps (v, x^0), we can define a Pareto improvement, or an *efficient trade* as λ^* such that

$$\lambda^* = \arg\max_{\lambda} \sum_i v_i(\lambda_i) \tag{4}$$

$$\text{s.t. } \lambda_{ij} + x_{ij}^0 \geq 0, \quad \forall i, \forall j \tag{5}$$

$$\sum_i \lambda_{ij} = 0, \quad \forall j \tag{6}$$

$$\lambda_{ij} \in \mathbb{Z}$$

Constraint (5) is used to enforce the cap of the mechanism in the volume of options traded, and in (6) the strict budget-balance is enforced through the free-disposal assumption. This would mean that the unwanted options could be freely allocated to traders. For example, if the bearish spread taker considers OTMs as substitutes, and pays only for the cheapest one, in case if the mechanism allocates the trader 2 OTM options, it is acceptable that the trader pays for only one option, and gets the other one for free.

It can be seen from the formulation of the efficient trade that the mechanism is aware of the caps for each trader. This is plausible because the traders participating in derivatives market maintain margin accounts which are continuously marked to the market by the broker as the market changes [8]. Hence the mechanism knows the capacity of each trader to issue options. Let us denote any feasible set of allocations for given x^0 as $\mathcal{F}(x^0)$, and let the any feasible set of allocations for trader i as $\mathcal{F}_i(x^0)$.

4.2 Bidding Language

is specifically designed for combinatorial exchanges by Cavallo et al. [3,11] and it can be used to express option portfolios as combinatorial bids to the mechanism. This language is fully expressive, and designed to be as concise and structured as possible. Because it is specifically designed for combinatorial exchange, it allows bidders to submit bids and asks simultaneously. The tree structure is used for expressing bids and asks connected through series of generalised logical connectives, such as 'interval-choose' (IC_x^y) operator. It specifies at least x and at most y of its child nodes must be satisfied. Hence, all intermediate nodes in Tree-based Bidding Language (TBBL) are IC_x^y nodes with corresponding price bounds. IC_x^y can replicate `OR` as IC_1^n, `XOR` as IC_1^1 and `AND` as IC_n^n. The leaf nodes of the TBBL are the actual bids or asks on options.

For example, trader is taking bullish spread, so he can submit following bid which generates positive cash flow if price goes up. Figure 2 shows how it can be represented in tree format, where each node has its own value, and plus sign implies bid, and minus ask. This shows that the trader is willing to buy call at strike \$90 for \$10, and (i.e. $IC(2,2)$) sell call at strike \$110 for \$5. We implicitly assume that the current asset price is $S_0 = \$100$, hence first call is ITM, and the second call is OTM. Similarly, the trader is also indifferent to buy the same bull spread with puts. Trader states that he wants to sell OTM put at strike \$90 for \$5 and buy ITM put at strike \$110 for \$10. Both spreads are evaluated at the price of \$5 for the trader.

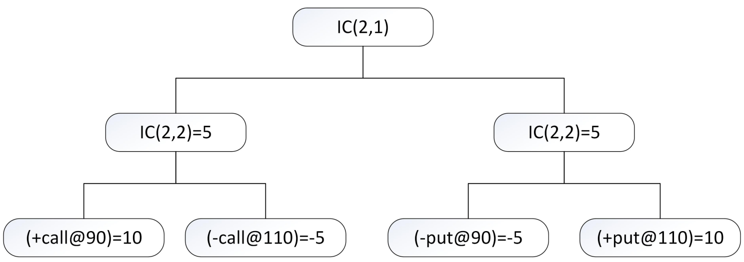

Fig. 2: Bullish spread expressed using TBBL

The important aspect of an above bidding structure displayed in Figure 2 is that the trader can express the bull spread both in terms of calls or puts. This gives him the flexibility in choosing equivalent option portfolios among possible allocations, and fully express his preferences to the mechanism. This is a great advantage of a combinatorial exchange over a double auction, because in this example bullish spread taker can fully reveal his indifference to trading various sets of options, as long as they make equivalent portfolios and generate the same payoff. In other words, instead of expressing his quotes to individual options as it was normally done in double auctions, in combinatorial exchange the trader is now capable of expressing his entire strategy to the mechanism. It is important to note that the combinatorial exchange is DSIC [2], and the traders are always better off revealing their true strategy to the mechanism. Complete revelation of trader's strategies consisting of option portfolios also provides more allocation choices to the mechanism.

Every trader i can submit bid T_i. Let $\theta \in T_i$ denote any node in the tree, and $v_i(\theta) \in \mathbb{R}$ is the value of this node for bidder i. A function $Leaf(T_i) \subset T_i$ returns all leaves of bid T_i, and $Child(\theta) \subset T_i$ returns all child nodes inside node θ. Any node θ is said to be satisfied by $IC_x^y(\theta)$ if:

- R1: Node θ with $IC_x^y(\theta)$ may be *satisfied* if only at least x and at most y of its children are satisfied.
- R2: If some node θ is not satisfied, then none of its children may be satisfied.

Let $G \in \{1, \ldots, m\}$ represents the options listed in the option chain. Let $\lambda_i \in \mathbb{Z}$ be the vector representing which option to take and which option to give for bidder i, or in other words a trade, or an allocation. Then the value v_i of the allocation λ_i for bid T_i is equal to the sum of all satisfied nodes. In order to represent satisfaction, let us define $sat_i(\theta) \in \{0, 1\}$ function which represents if $\theta \in T_i$ is satisfied. Valid set of solutions for T_i can be derived through applying R1 and R2 to all internal nodes of T_i, such that $\theta \in \{T_i \backslash Leaf(T_i)\}$. Hence for $IC_x^y(\theta)$ following condition should hold:

$$x sat_i(\theta) \leq \sum_{\theta' \in Child(\theta)} sat_i(\theta') \leq y sat_i(\theta) \tag{7}$$

Secondly, we also do not want for any given trade λ, the number of options supplied is less than demanded. So we can write this constraint as follows:

$$\sum_{\theta \in Leaf(T_i)} sat_i(\theta) \leq \lambda_{ij}, \forall j \tag{8}$$

So the rules R1 (7) and R2 (8) form the validity function $valid(T_i, \lambda_i)$ for a given bid tree T_i and allocation λ_i. This validity function returns the mapping for the satisfiability of each node θ in T_i under given λ_i. Hence we can right the satisfiability function as $sat_i \in valid(T_i, \lambda_i)$.

The valuation of the given tree T_i holds free disposal rule where unsold options can be freely allocated to any bidder. With given constraints we can formulate a valuation function for given bid T_i,

$$v_i(T_i, \lambda_i) = \max_{sat_i} \sum_{\theta \in T_i} v_i(\theta) sat_i(\theta) \tag{9}$$

$$\text{s.t. (7) (8)} \tag{10}$$

[2] See Parkes et al. [11] for proof.

4.3 Winner Determination

We formulate in the form of an Integer Linear Programme (ILP) using the TBBL for a combinatorial exchange. This finds the efficient set of allocations λ for the given capacity x^0 in the combinatorial exchange. Let us define $T = (T_1, \ldots, T_n)$ as the TBBL bids submitted to the mechanism. Also let us denote the tree node $\theta \in \lambda_i$ if $\theta \in T_i$ and it is satisfied by trade λ_i written as $sat_i(\theta) = 1$. Then we can formulate the Winner Determination Problem (WDP) for the option exchange as shown below:

$$WD(T, x^0) = \lambda^* = \arg\max_{\lambda, sat} \sum_i \sum_{\theta \in T_i} v_i(\theta) sat_i(\theta) \tag{11}$$

$$\text{s.t. (5), (6)}$$

$$sat_i \in valid(T_i, \lambda_i), \forall i \tag{12}$$

$$sat_i(\theta) \in \{0, 1\}, \lambda_{ij} \in \mathbb{Z} \tag{13}$$

The solution of the above WDP should give the matrix of allocations λ that maximise the surplus for posted options. The mechanism has to choose the valid mappings for sat_i for the nodes of submitted tree bids T and at the same time maximise the valuations of these nodes.

4.4 Threshold Payments

We formulate payments rule as an optimisation problem using the minimisation of the worst difference in Vickrey-Clarke-Groves (VCG) payments and threshold payments. The threshold payment rule for obtaining budget-balanced VCG payments is given in (14). Also let λ^*_{-i} be the combinatorial exchange's allocation of options where trader i did not participate.

$$\rho_{vcg,i} = v_i(\lambda^*_i) - \Delta_{vcg,i} \tag{14}$$

$$\Delta_{vcg,i} = \left(\sum_j v_j(\lambda^*_j) - \sum_{j \neq i} v_j(\lambda^*_{-i,j}) \right) \tag{15}$$

where $\rho_{vcg,i}$ is the VCG payment for the trader i, and $\Delta_{vcg,i}$ is called as the VCG discount for the trader i. Also note that the trader produces a scalar valuation for the given allocation vector λ^*_i, as the trader values the bundle as a whole. Then we can find such $\Delta_{thresh,i}$ which solves the minimisation of the worst difference between VCG discount and the threshold discount.

$$\boldsymbol{\Delta}^*_{thresh} = \arg\min_{\boldsymbol{\Delta}_{thresh}} \epsilon \tag{16}$$

$$\text{s.t. } \Delta_{vcg,i} - \Delta_{thresh,i} \leq \epsilon \quad \forall i \tag{17}$$

$$\Delta_{vcg,i} - \Delta_{thresh,i} \geq 0 \quad \forall i \tag{18}$$

$$\sum_i \Delta_{thresh,i} \leq \sum_i v_i(\lambda^*_i) \tag{19}$$

The solution $\boldsymbol{\Delta}^*_{thresh}$ can be used to compute the budget-balanced payments for traders given formula below:

$$\rho_{thresh,i} = v_i(\lambda^*_i) - \Delta_{thresh,i} \quad (20)$$

To illustrate the combinatorial exchange in option market, consider following example given in Figure 3. There are two traders who submitted their bids to the combinatorial exchange to take a corresponding position in the option market. First trader is bullish trader because he is willing to buy a bullish spread. Although he specified which ITM call he wants, he is indifferent for the OTM call he wants to sell. So he can sell either call at strike $\$105$ for \$4, or call at strike $\$110$ for \$2. Note that here bullish trader is regarding the OTM calls as substitutes. The second trader is bearish in his belief on asset prices, so he expects a asset prices to go down, and wants to take a bearish spread. So he submits one ask for the ITM call, and he is indifferent in buying either of OTM calls. There are 2 potential matches in given example. In the first match, bullish trader

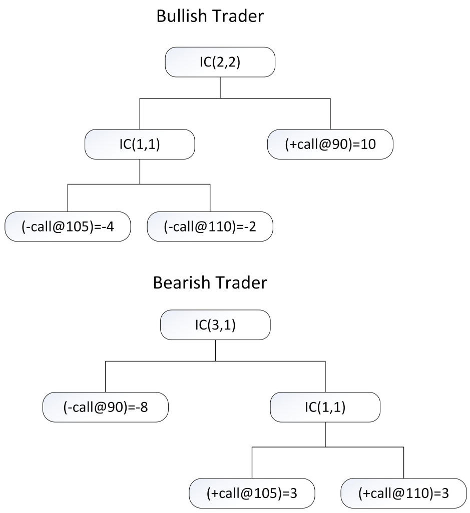

Fig. 3: Bullish and Bearish traders' bids using TBBL

is going to buy ITM call from bearish trader creating a surplus of \$2, and sell OTM call at strike $\$105$ for \$4 to the bearish trader creating a deficit of \$1. In the second case, bullish trader does the same with the ITM call, but sells OTM call at strike $\$110$ for \$2 creating a surplus of \$1. The surplus maximising allocation would choose the second matching, as it maximises the overall surplus to \$3.

Now let us consider the payments paid and received by both traders using threshold rule. Considering it using VCG scheme, we understand that the removal of any trader would cause null trade. Hence the VCG discount of both traders is equal to the surplus made, which is \$3. In this way, the bullish trader's total payment to the mechanism is $\$10 - \$2 - \$3 = \$5$. However the bearish trader's total payment to the mechanism is $-\$8 + \$3 - \$3 = -\$8$, so he should receive \$8 from the mechanism which runs into deficit of $\$5 - \$8 = -\$3$. But now using the threshold rule, we can find the minimised deviance of the discounts from the VCG discounts. In this example, the threshold discount is $\$1.5$. So applying the threshold discount instead of VCG discounts, we get a payment from bearish trader $\$10-\$2-\$1.5 = \$6.5$ and get a payment from the bullish trader $-\$8 + \$3 - \$1.5 = -\$6.5$. As it can be seen mechanism balances the deficit, through decreasing the amount of the VCG discount for both traders.

5 More on TBBL

We can further adapt TBBL for expressing option portfolios. This would require the separation of concepts of option pricing and preferences specification. Although it is common for existing bidding languages to combine both concepts into single atomic bid, in option portfolio market it becomes redundant to provide valuations for the identical goods appearing in different portfolios, because we need to enforce the consistency of option prices throughout multiple portfolios. The mechanism can also be easily checked if the individual option prices are within their legal bounds (i.e. does not incur guaranteed loss). Moreover there is possibility for the trader just to disclose his belief and the number of options in his portfolio, and the mechanism can automatically pick the right option portfolio for him.

The trader sends to the mechanism the list of his private valuations for the options and his preference over these options expressed through TBBL. While referencing the options in the TBBL, the trader does not indicate his valuation, but just indicate the option type and the strike he wishes to buy or sell. This requirement is necessary in option market because the mechanism should be able to check the consistency of option prices. For above reasons the traders are required to declare their valuations of individual options to the mechanism.

On the other hand, the traders can be freed from the burden of building the TBBL to express their preferences. For instance, consider a bullish trader who wants to take a bullish spread, but he has not clarified which concrete ITM call and OTM call he wants to buy and sell. He can submit to the mechanism following generic TBBL shown in Figure 4 along with his option valuations for each option in the option chain. Hence the trader declares all of his valuations in one list, and then interprets his generic bullish preference to the mechanism. The options that the trader would like to use are highlighted with shaded background in Figure 4. Note that the trader did not give his valuations inside TBBL, but sent them in a separate list. This generic and declarative approach can be used for other types of options portfolios which contain more than two options. For instance, the neutral option portfolios such as butterfly, long ladder, etc contain more than two options, and can also be combined into generic format in the same way it was done with spreads.

6 Concluding Remarks

In this paper, we described potential application of combinatorial exchanges in option market and discussed its advantages. We described the possible cases where the substitutability and complementarity relationship between different options may emerge.

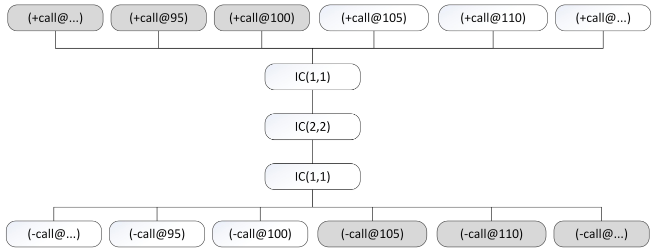

Fig. 4: Generic template for double-option spreads using TBBL

We used the Parkes et al. 's ICE to accommodate options with different moneyness into pricing process. Morever, we presented the further improvements by changing the structure of TBBL. We also introduced a generic class of TBBLs that could be used to represent any kind of option portfolio with no specification on the strike prices.

However, there are still many aspects of the research yet to be studied. Research needs an empirical analysis with real data from option markets to verify its capacity to approximate the prices of any given option portfolio. This requires simulation of the market with benchmark trading strategies such ZI-C [6] and ZIP [4]. This would entail the development of online combinatorial exchanges which could better adapt to current financial exchanges.

References

1. Bates, D.S.: Empirical option pricing: A retrospection. Journal of Econometrics 116(1), 387–404 (2003)
2. Black, F., Scholes, M.: The pricing of options and corporate liabilities. The journal of political economy pp. 637–654 (1973)
3. Cavallo, R., Parkes, D., A.I, J., Kirsch, A., Kulesza, A., Lahaie, S., Lubin, B., Michael, L., Shneidman, J.: Tbbl: A tree-based bidding language for iterative combinatorial exchanges. Tech. rep., Harvard University (2005)
4. Cliff, D.: Minimal-intelligence agents for bargaining behaviours in market-based environments. Tech. rep., Hewlett-Packard Research Lab (1997)
5. Crampton, P., Shoham, Y., Steinberg, R.: Combinatorial Auctions. MIT Press (2010)
6. Gode, D., Sunder, S.: Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality. Journal of Political Economy 101-1, 119–137 (1993)
7. Heston, S.L.: A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of financial studies 6(2), 327–343 (1993)
8. Hull, J.C.: Options, futures, and other derivatives. Prentice Hall, 8 edn. (2012)
9. Merton, R.C.: Theory of rational option pricing. The Bell Journal of economics and management science pp. 141–183 (1973)
10. Nisan, N.: Bidding and allocation in combinatorial auctions. In: In Proc. 2nd ACM Conf. on Electronic Commerce(EC-00). pp. 1–12 (2000)
11. Parkes, D.C., Lubin, B., Juda, A., Cavallo, R., Lahaie, S., Shneidman, J.: Ice: An expressive iterative combinatorial exchange. Journal of Artificial Intelligence Research 33, 33–77 (2008)
12. Roughgarden, T.: Lectures on combinatorial auctions. lecture notes on topics in algorithmic game theory (2008)