# A Study on Visualization Methods of Phyllotaxis Pattern

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## Abstract

In this paper we introduce some softwares developed that is related to the modeling of phyllotaxis of plant or flowers. Based on the theories and applications of these research area, we reviewed visualization software which can be used to find match flower pattern especially focus on spiral phyllotaxis like sunflower. Using the patterns that we have derived from presented programs, we can draw the classification of flowers according to respective parameters. This could be handy in learning the mathematical diversity of different flower types and could help to find the similarities in the nature of flower pattern structure.

## 1. Introduction

It is well known that Fibonacci numbers appear frequently in plants. This has resulted in many applications of mathematics to plants such as spiral phyllotaxis. Recently, improvements in experimental techniques have increased our understanding of plant development. Also high speed computers have given us a better understanding of the complexities in the developmental process. In nature majority of plants reflect Fibonacci phyllotaxis pattern, featuring Fibonacci numbers and the Golden Angle. The Golden Angle is related to the Golden Mean, itself a limit of quotients of Fibonacci numbers.

Golden mean : $\phi = (1+\sqrt{5})/2 = 1.61803...$

Golden angle : $360°(2-\phi) \cong 137.51°$

We will review these facts below. Based on a survey of the literature encompassing 650 species and 12500 specimens, Jean [3] estimated that, among plants displaying spiral or multijugate phyllotaxis about 92% of them have Fibonacci phyllotaxis. In 1202 Leonardo Fibonacci obtained the Fibonacci sequence <1, 1, 2, 3, 5, ... , F(n), F(n+1)> as a solution to the problem of monthly population growth of rabbits. This sequence relates to $\phi$ by the formula lim [F(n+1)/F(n)] = $\phi$. To find out more about the Fibonacci numbers and the Golden Mean, visit Knott's web site [2].

In 1948 and 1951 Richards introduced the idea of the plastochrone ratio and developed a system of equations to mathematically describe a centric representation using three parameters: plastochrone ratio, divergence angle, and the angle of the cone tangential to the apex in the area being considered. The field of dynamical systems has developed powerful technique for understanding complex process. The dynamical model offers an explanation of why Fibonacci phyllotaxis is so predominant. For a brief history of phyllotaxis research, refer to http://maven.smith.edu/ ~phyllo/OldFiles/History/historynoroll.html.

## 2. Spiral Phyllotaxis

The phyllotaxis spiral exactly maps with Fermat's spiral which is also referred as parabolic spiral. If we place the meshes of phyllotaxis along spiral according to Fibonacci pattern and use certain angle while defining the position of next mesh in respect to previous mesh, we will get the very phyllotaxis spiral. We can visualize the phyllotaxis spiral in two ways: first is the planar view and the second is the cylindrical view.

The planar view of phyllotaxis can comprehensively illustrate the sunflower head or daisy. Vogel [6] proposes the mathematical model for planar spiral of phyllotaxis.

$$\varphi = n \times 137.5°, \qquad r = c\sqrt{n}$$

where:

- $n$ is the ordering number of a floret, counting outward from the center. This is the reverse of floret age in a real plant.
- $\varphi$ is the angle between a reference direction and the position vector of the nth floret in a polar coordinate system originating at the center of the capitulum. It follows that the divergence angle between the position vectors of any two successive florets is constant, α = 137.5°.
- $r$ is the distance between the center of the capitulum and the center of the nth floret, given a constant scaling parameter c.


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Also Vogel [6] assumes that each new floret is issued at a fixed angle α with respect preceding floret and the position vector of each new floret fits into the largest gap between the position vectors of the older florets.

These assumptions have been criticized by Ridley [7], so he states that it is reasonable to assume that the plant could contain genetic information determining the divergence angle to some extent, so it is completely impossible for this alone to fix the divergence angle to the incredible accuracy occurring in nature, since natural variation in biological phenomena is normally rather wide. For example, for the 55 and 89 parastichies to be conspicuous, as occurs in most sunflower heads, divergence angle must lie between 21/55 and 34/89. So it is necessary to highlight that there is no fixed angle repeating the pattern of seeds in sunflower. It can vary within some boundaries.

In this paper, we will illustrated the four main types of phyllotaxis such as Distichous Phyllotaxis, Whorled Phyllotaxis, Spiral Phyllotaxis, Multijugate Phyllotaxis. They can be detected and further classified by the number of visible spirals (parastichies) they display. There are two parastichies in planar phyllotaxis. The number of parastichies can be from 21 and 34 for a small capitulum, up to 89 and 144 or even 144 and 233 for large ones. For example, the capitulum of a daisy exhibits 34 clockwise spirals and 21 counterclockwise spirals, with directions determined by following the spirals outwards from the capitulum center. The domestic sunflower capitulum, one can discern 34 spirals running clockwise and 55 spirals running counterclockwise. So it is possible to classify the flowers programmatically through detecting the number of parastichies they possess. All these patterns can be modeled by simple lattice-like mathematical structures. Spiral phyllotaxis is the most frequent. Other patterns may exist that are not quite as regular, and seldom mentioned by botanists.

## 2.1 Phyllotaxis : Botanical Spatial

Phyllotaxis is the study of the successive arrangement of radial parts of a growing plant, such as the leaves on a stem, or the seeds in the head of a sunflower. Some examples from nature can be seen here. Essentially, these arrangements can be generated using the following simple procedure. Imagine the first leaf coming off from the stem of some plant. The second leaf that grows from the stem will, first of all, be further outwards or further up the stem, and secondly, it will grow from the stem at a different angle than the first leaf. The third leaf then does the same as the second. It grows further outwards or further up the stem than the second did, and the angle at which it grows has increased again by the same amount it did between the first and the second leaves. That is the procedure for generating a phyllotaxic arrangement of leaves, or other botanic evolutions.

The program developed by Brian DiLoreto [8] have two controls "locus angular separation" and "locus radial separation". It also provides the result of space density to the radius of density sample. The program has been developed using C++ programming language which uses graphical libraries in order to visualize the pattern.

This application generates a simple phyllotaxic arrangement of circles, expanding outward at either a linear or an exponential rate. You can specify the angle between each circle (between 0 and 360 degrees), and you can determine rate of expansion using a pair of slider controls. Animated transitions between phyllotaxis arrangements which differ in their angle, or their exponential rate, or both, can be created using another corresponding pair of buttons to the left of the slider controls for angle and expansion rate.

Furthermore, you can print an image of any phyllotaxis arrangement of any size. And graphs can be displayed showing the radius of each of the successive circles in the phyllotaxic arrangement, or showing the density of space (measured as the fraction of space which is contained in the interior of the circles; that is, the fraction of space which is occupied by the area of the circles) as the radius of the density sample increases. We can also apply this idea to find the optimal space for memory storage in computer.

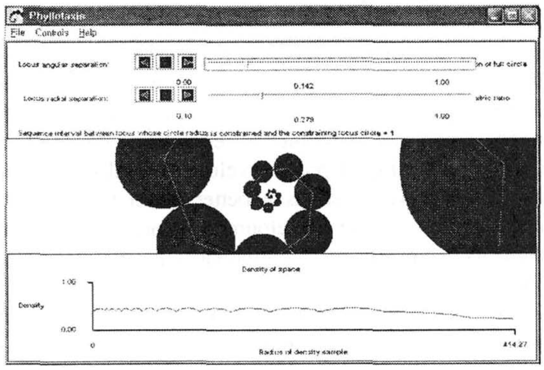


**Figure 1: Botanical Spatial Visualization Software**

## 2.2 Spiral Lattices

Spiral lattices arrangement of points on concentric circles with radius increasing at a constant rate and with constant (divergence) angle between successive points can be obtained as the set of integer powers of a single complex number. Spiral lattices model are configurations of points placed on concentric circles such that:

- There is one point on each circle.

- The divergence angle ($d$) between points on successive circles is constant.
- The radius of successive concentric circles increases by a constant factor $G$, the expansion parameter (also called Plastochrone Ratio).

The coordinates of the $k$th point in a spiral lattice with divergence $d$ and expansion parameter $G$ are given by ($G^k\cos(kd)$, $G^k\sin(kd)$). Note that these points all belong to a continuous spiral parametrized by ($G^t\cos(td)$, $G^t\sin(td)$), where $t$ is a real number. This spiral is called the generative spiral. The generative spiral is usually not the one that is visible: in most cases, it winds too much to catch the eye. Certain patterns (eg. the head of a sunflower) are better approximated by (sqrt($kG$)cos($kd$), sqrt($kG$) sin($kd$)).

This spiral program, written at different stages by Scott Hotton (UCSC), Pau Atela, Chris Golé and Biliana Kaneva (Smith College) [1], explores the universe of spiral lattices. The program shows all mathematically possible spiral lattices and the underlying structure that classifies them. It then helps the user identify those lattices that are favored by plants (Fibonacci phyllotaxis). The program has been developed in Java, which is considered to platform-independent architecture and can run on any operating system. Java possess rich library of computer graphics named as AWT which makes the visualization of phyllotaxis pattern very easy.

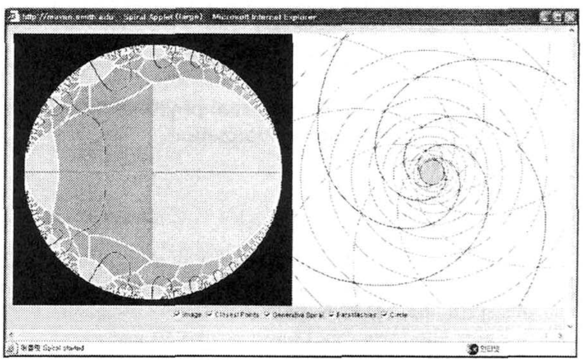

**Figure 2: Spiral Lattice Java Applet Program**

## 2.3 Cylinder Lattice Applet

This applet shows all the possible (half) cylindrical lattices. Think of the right window as a vertical cylinder (e.g. the stem of a plant) which has been unrolled. In terms of botany, the vertical axis is the internodal distance, the horizontal axis is the divergence angle. Note that, by symmetry, only positive divergence angles are represented.

The Cylinder Lattice could be helpful while illustrating the evolution of leaves on stem. It could be handy in finding the pattern of leaf growth of particular plant. For example, the pineapple could be the apparent illustration of cylindrical phyllotaxis pattern. This software program has also been built using Java technology which provides enough functions to display the cylindrical phylotaxis pattern.

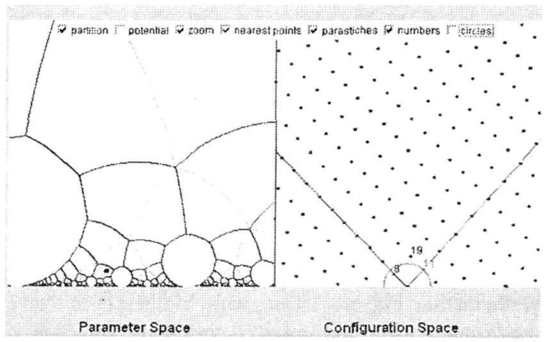


**Figure 3: Cylinder Lattice Java Applet Program**

## 2.4 Dynamical System Model

The field of dynamical systems has developed powerful technique for understanding complex process. Hotton [4,5] defined and investigated two families of dynamic systems that model meristematic plant development. The dynamic systems approach goes beyond merely providing a mathematical framework for describing the patterns observed in plants and can be used to make testable predictions about the development process.

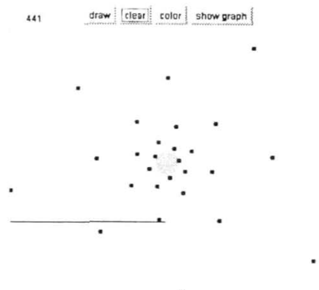


**Figure 4: Dynamic System Model Program**

The model used in dynamic system model is a discrete dynamical system. Generally, a discrete dynamical system consists of iterating a function (also called map) F on a set S (also called the state space). In this model, each element in S corresponds to a configuration of primordia, which are represented by points on concentric circles, with one


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point per circle. The innermost circle represents the periphery of the meristem. From circle to circle the radius increases by a constant factor *G*, that we call expansion rate (Plastochrone ratio in the literature). Each configuration is encoded as a list of divergence angles between successive points. Hence, S can be seen as the set of all possible lists $(d_1, d_2, \ldots, d_N)$ of angles.

### 2.5 Wolfram Demonstrations Project

The Wolfram Demonstrations Project is an open-code resource that uses dynamic computation to illuminate concepts in science, technology, mathematics, art, finance, and a remarkable range of other fields. Interactive illustrations are created by Mathematica. All Demonstrations run freely on any standard Windows, Mac, Unix, or Linux computer. In fact, you don't even need Mathematica to benefit. Anyone can preview a Demonstration online, and interact with it using the free Mathematica Player. Those with Mathematica can also experiment with and modify the code on their own computers [9].

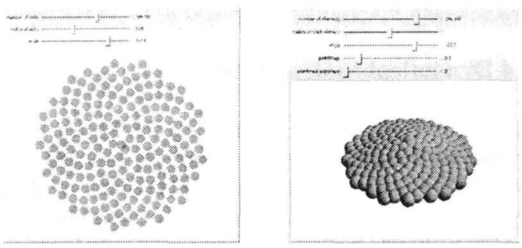

**Figure 5: Example of Phyllotaxis (Mathematica)**

The following source code can be found in "Phyllotaxis Spirals in 3D" from The Wolfram Demonstrations Project [9].

```
Manipulate[
  Graphics3D[
   {Pink,
    Table[
    Sphere[Sqrt[i] {Sin[i (a Degree)], Cos[i (a
          Degree)], -p N[i+1^-10]^e}, size], {i, 0, n}]},
    ImageSize {450, 300}, Boxed→False,
    PlotRange→{{-Sqrt[n]-size, Sqrt[n]+size}, {-
          Sqrt[n]-size, Sqrt[n]+size}, All}],
   {{n, 100, "number of elements"} ,1,250,
    Appearance→"Labeled"},
   {{size, 1.5, "radius of each element"}, 0.1, 3},
   {{a, 137.5, "angle"}, 0,180, Appearance   "Labeled"},
   {{p, 0.3, "pointiness"}, 0, 2, Appearance   "Labeled"},
   {{e, 0, "pointiness exponent"}, 0, 0.65,
   Appearance→"Labeled"}]
```

## 3. Conclusion and Future Works

We reviewed several basic concepts of mathematical modeling of phyllotaxis pattern. Also in this paper, we presented the two visualization types of phyllotaxis spiral: planar and cylindrical views. Moreover the possibility of programmatic classification of flowers and using the parameterized attributes of phyllotaxis pattern helped us to identify the exclusive properties of daisy and sunflower heads. At last, we reviewed several software applications which simulate the visualization of phyllotaxis pattern both in planar and cylindrical views. The visualization software of phyllotaxis is very useful tool for catching the taxonomy between natural species of phyllotaxis flowers and their virtual visualization.

So in future, we are looking forward to continue the improvement of this concept. We aimed to create the catalog of classified flowers according the parametric measurements of phillotaxis spiral. In our next research works, we also put our effort on developing a pattern matching algorithm which will be able to recognize the phylotaxis spiral of a given flower capitulum. The idea behind our proposal is shown in figure below:

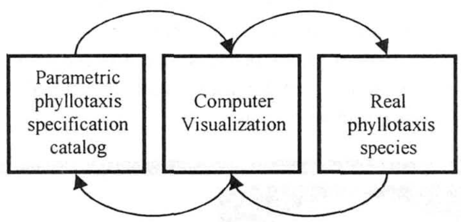


**Figure 6: The recognition of real phyllotaxis species through visualization**

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